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$D_{sJ}(2860)$ and $D_{sJ}(2715)$

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Abstract. Recently the Babar Collaboration reported a new $c\bar{s}$ state, $D_{sJ}(2860)$, and the Belle Collaboration observed $D_{sJ}(2715)$. We investigate the strong decays of the excited $c\bar{s}$ states using the $^3P_0$ model. After comparing the theoretical decay widths and decay patterns with the available experimental data, we are inclined to conclude that: (1) $D_{sJ}(2715)$ is probably the $1^-(1\,^3D_1)$ $c\bar{s}$ state, although the $1^-(2\,^3S_1)$ assignment is not completely excluded; (2) $D_{sJ}(2860)$ seems unlikely to be the $1^-(2\,^3S_1)$ and $1^-(1\,^3D_1)$ candidate; (3) to consider $D_{sJ}(2860)$ either as a $0^+(2\,^3P_0)$ or as a $3^-(1\,^3D_3)$ $c\bar{s}$ state is consistent with the experimental data; (4) the experimental search of $D_{sJ}(2860)$ in the channels $D_s\eta$, DK^*, D^*K and $D_s^*\eta$ will be crucial to distinguish the above two possibilities.

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1 Introduction

Recently, the Babar Collaboration observed a new $c\bar{s}$ state, $D_{sJ}(2860)$, with a mass $2856.6\pm1.5\pm5.0$ MeV and a width $\Gamma=(48\pm7\pm10)$ MeV. Babar observed it only in the D^0K^+, $D^+K_S^0$ channels and found no evidence for $D^{*0}K^+$ and $D^{*+}K_S^0$. Thus, we have $J^P=0^+, 1^-, 2^+, 3^-, \ldots$. At the same time, the Belle Collaboration reported a broader $c\bar{s}$ state, $D_{sJ}(2715)$, with $J^P=1^-$ in $B^+\to\bar{D}^0D^0K^+$ decay [2]. Its mass is $2715\pm11^{+11}_{-14}$ MeV and its width $\Gamma=(115\pm20^{+36}_{-32})$ MeV.

$D_{sJ}(2860)$ was proposed to be the first radial excitation of $D_{sJ}^*(2317)$ in [3, 4], as a $J^P=3^-$ $c\bar{s}$ state in [5] and as a $c\bar{s}(2P)$ state in [6]. $D_{sJ}(2715)$ sits exactly on the quark model prediction 2720 MeV for the $2\,^3S_1$ $c\bar{s}$ state [7]. The 1^- state lies around 2721 MeV if one requires the $(1^+,1^-)$ $c\bar{s}$ states to form a chiral doublet [8].

According to the heavy quark effective field theory, heavy mesons form doublets. For example, we have one s-wave $c\bar{s}$ doublet $(0^-,1^-)=(D_s(1965), D_s^*(2115))$ and two p-wave doublets $(0^+,1^+)=(D_{sJ}^*(2317), D_{sJ}(2460))$ and $(1^+,2^+)=(D_{s1}(2536), D_{s2}(2573))$. The two d-wave $c\bar{s}$ doublets $(1^-,2^-)$ and $(2^-,3^-)$ have not been observed yet.

The possible quantum numbers of $D_{sJ}(2860)$ include $0^+(2\,^3P_0)$, $1^-(1\,^3D_1)$, $1^-(2\,^3S_1)$, $2^+(2\,^3P_2)$, $2^+(1\,^3F_2)$ and $3^-(1\,^3D_3)$. The $2\,^3P_2$ $c\bar{s}$ state is expected to lie around (2.95–3.0) GeV, while the mass of the $1\,^3F_2$ state will be much higher than 2.86 GeV. In the following we focus on the $0^+, 1^-, 3^-$ assignments.

In this work, we will try different assignments for $D_{sJ}(2860)$ and $D_{sJ}(2715)$ and investigate their different decay patterns and total widths within the framework of the $^3P_0$ model. After comparing the theoretical total width and the decay patterns with the available experimental information, one may get a hint of their favorable quantum numbers and the assignments in the quark model.

This paper is organized as follows. We give a brief review of the $^3P_0$ model in Sect. 2. Next we present the strong decay amplitudes of various low-lying excited $c\bar{s}$ states in Sects. 3–6. We present our numerical results in Sect. 7. In the last section is the discussion. We collect some lengthy formulae in the appendix.

2 The $^3P_0$ model

The $^3P_0$ model was first proposed by Micu [9] and further developed by Yaouanc et al. later [11–16]. This model has been widely used to study the hadron decay widths [17–24]. According to this model, a pair of quarks with $J^{PC}=0^{++}$ is created from the vacuum when a hadron decays, which is shown in Fig. 1 for the meson decay process $A\to BC$. The new $q\bar{q}$ pair created from the vacuum together with the $q\bar{q}$ within the initial meson regroups into the outgoing mesons via the quark rearrangement process. In the nonrelativistic limit, the transition operator is written as

$$T=-3\gamma\sum_m\langle 1m;1-m|00\rangle(2\pi)^{\frac{3}{2}}\times\int \mathrm{d}^3\mathbf{k}_3\,\mathrm{d}^3\mathbf{k}_4\,\delta^3(\mathbf{k}_3+\mathbf{k}_4)\mathcal{Y}_1^m(\mathbf{k}_3-\mathbf{k}_4)\times\chi^{34}_{1,-m}\varphi^{34}_0\omega^{34}_0 b^\dagger_{3i}(\mathbf{k}_3)d^\dagger_{4j}(\mathbf{k}_4)\,, \quad (1)$$

where i and j are the SU(3) color indices of the created

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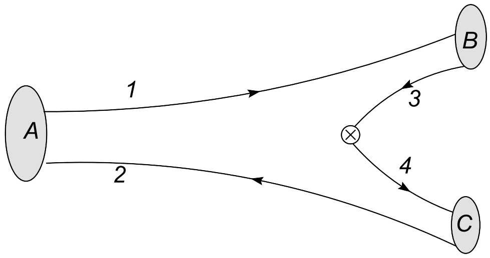


Fig. 1. The decay process $A \to BC$ in the $^3P_0$ model

quark and antiquark. $\varphi_0^{34} = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $\omega_0^{34} = \delta_{ij}$ are for the flavor and color singlets, respectively. $\chi_{1,-m}^{34}$ is a spin triplet state. $\mathcal{Y}_1^m(\mathbf{k}) \equiv |\mathbf{k}|^l Y_l^m(\theta_k, \phi_k)$ is a solid-angle harmonic polynomial corresponding to the p-wave quark pair. γ is a dimensionless constant that denotes the strength of quark pair creation from the vacuum and can be extracted by fitting data. The meson state is defined by [25]

$$\begin{aligned} &\left|A\left(n_A{}^{2S_A+1}L_{A J_A M_{J_A}}\right)(\mathbf{p}_A)\right\rangle \\ &= \sqrt{2E_A} \sum_{M_{L_A}, M_{S_A}} \langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A} \rangle \\ &\times \int \mathrm{d}^3\mathbf{k}_1\, \mathrm{d}^3\mathbf{k}_2 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{p}_A) \\ &\times \psi_{n_A L_A M_{L_A}}(\mathbf{k}_1, \mathbf{k}_2) \chi_{S_A M_{S_A}}^{12} \varphi_A^{12} \omega_A^{12} |q_1(\mathbf{k}_1) \bar{q}_2(\mathbf{k}_2)\rangle \end{aligned} \tag{2}$$

and satisfies the normalization condition

$$\langle A(\mathbf{p}_A) | A(\mathbf{p}'_A) \rangle = 2E_A \delta^3(\mathbf{p}_A - \mathbf{p}'_A)\,. \tag{3}$$

The subscripts 1 and 2 refer to the quark and antiquark within the parent meson A, respectively. $\mathbf{k}_1$ and $\mathbf{k}_2$ are the momenta of the quark and antiquark. $\mathbf{k}_A$ is their relative momentum and $\mathbf{p}_A$ is the momentum of meson A. $S_A = s_{q_1} + s_{q_2}$ is the total spin. $J_A = L_A + S_A$ is the total angular momentum.

The S-matrix is defined by

$$\langle f|S|i\rangle = I + \mathrm{i}(2\pi)^4 \delta^4(p_f - p_i) \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}\,. \tag{4}$$

The decay helicity amplitude of the process $A \to B + C$ in the center of mass frame of meson A is

$$\begin{aligned} &\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(A \to BC) \\ &= \sqrt{8E_A E_B E_C}\gamma \sum_{\substack{M_{L_A}, M_{S_A}, \\ M_{L_B}, M_{S_B}, \\ M_{L_C}, M_{S_C}, m}} \langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A} \rangle \\ &\times \langle L_B M_{L_B} S_B M_{S_B} | J_B M_{J_B} \rangle \langle L_C M_{L_C} S_C M_{S_C} | J_C M_{J_C} \rangle \\ &\times \langle 1m; 1-m|00\rangle \langle \chi_{S_C M_{S_C}}^{32} \chi_{S_B M_{S_B}}^{14} | \chi_{S_A M_{S_A}}^{12} \chi_{1-m}^{34} \rangle \\ &\times \langle \varphi_C^{32} \varphi_B^{14} | \varphi_A^{12} \varphi_0^{34} \rangle I_{M_{L_B}, M_{L_C}}^{M_{L_A}, m}(\mathbf{p})\,, \end{aligned} \tag{5}$$

where the spatial integral $I_m(A, BC)$ is defined as

$$\begin{aligned} I_{M_{L_B}, M_{L_C}}^{M_{L_A}, m}(\mathbf{p}) &= (2\pi)^{3/2} \int \mathrm{d}^3\mathbf{k}_1\, \mathrm{d}^3\mathbf{k}_2\, \mathrm{d}^3\mathbf{k}_3\, \mathrm{d}^3\mathbf{k}_4 \\ &\times \delta^3(\mathbf{k}_3 + \mathbf{k}_4) \delta^3(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{p}_A) \\ &\times \delta^3(\mathbf{k}_1 + \mathbf{k}_3 - \mathbf{p}_B) \delta^3(\mathbf{k}_2 + \mathbf{k}_4 - \mathbf{p}_C) \\ &\times \psi_{n_B L_B M_{L_B}}^*(\mathbf{k}_1, \mathbf{k}_3) \psi_{n_C L_C M_{L_C}}^*(\mathbf{k}_2, \mathbf{k}_4) \\ &\times \psi_{n_A L_A M_{L_A}}(\mathbf{k}_1, \mathbf{k}_2) \mathcal{Y}_1^m\left(\frac{\mathbf{k}_3 - \mathbf{k}_4}{2}\right). \end{aligned} \tag{6}$$

The spin matrix element can be written in terms of Wigner's $9j$ symbols [16]

$$\begin{aligned} &\langle \chi_{S_C M_{S_C}}^{32} \chi_{S_B M_{S_B}}^{14} | \chi_{S_A M_{S_A}}^{12} \chi_{1-m}^{34} \rangle \\ &= \sum_{S_{BC}, M_s} \langle S_C, M_{S_C}; S_B, M_{S_B} | S, M_s \rangle \\ &\times \langle S, M_s | S_A, M_{S_A}; 1, -m \rangle \\ &\times [3(2S_B+1)(2S_C+1)(2S_A+1)]^{1/2} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & S_C \\ \frac{1}{2} & \frac{1}{2} & S_B \\ S_A & 1 & S \end{Bmatrix}. \end{aligned}$$

The relevant flavor matrix element is

$$\begin{aligned} \langle \varphi_C^{32} \varphi_B^{14} | \varphi_A^{12} \varphi_0^{34} \rangle &= \sum_{I, I^3} \langle I_C, I_C^3; I_B I_B^3 | I_A, I_A^3 \rangle \\ &\times [(2I_B+1)(2I_C+1)(2I_A+1)]^{1/2} \\ &\times \begin{Bmatrix} I_1 & I_3 & I_C \\ I_2 & I_4 & I_B \\ I_A & 0 & I_A \end{Bmatrix}, \end{aligned}$$

where I_i $(i = 1, 2, 3, 4)$ is the isospin of quark, which is labeled in Fig. 1.

With the Jacob–Wick formula the helicity amplitude can be converted into the partial wave amplitude [26]:

$$\begin{aligned} \mathcal{M}^{JL}(A \to BC) &= \frac{\sqrt{2L+1}}{2J_A+1} \\ &\times \sum_{M_{J_B}, M_{J_C}} \langle L, 0; J, M_{J_A} | J_A, M_{J_A} \rangle \\ &\times \langle J_B, M_{J_B}; J_C, M_{J_C} | J, M_{J_A} \rangle \\ &\times \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\mathbf{k}_B)\,, \end{aligned} \tag{7}$$

where $M_{J_A} = M_{J_B} + M_{J_C}$, $\mathbf{J} = \mathbf{J}_B + \mathbf{J}_C$ and $\mathbf{J}_A + \mathbf{J}_P = \mathbf{J}_B + \mathbf{J}_C + \mathbf{L}$. The decay width in terms of the partial wave amplitudes using the relativistic phase space is

$$\Gamma = \frac{1}{8\pi} \frac{|\mathbf{p}|}{M_A^2} \sum_{JL} |\mathcal{M}^{JL}|^2,$$

where $|\mathbf{p}|$ is the three momentum of the daughter meson in the parent's center of mass frame.

3 Strong decays of $0^+(2\,^3P_0)$ $c\bar{s}$ state

3.1 D^0K^+, D^+K^0, $D_s^+\eta$ modes

The harmonic oscillator wavefunctions of $0^+(2\,^3P_0)$ $c\bar{s}$ state can be expressed as

$$\psi^{n=2;L=1}(\mathbf{p}_1,\mathbf{p}_2)=\frac{\mathrm{i}R^{5/2}}{\sqrt{15}\pi^{1/4}}\left[10-R^2(\mathbf{p}_1-\mathbf{p}_2)^2\right] \times\mathcal{Y}_1^m\left(\frac{\mathbf{p}_1-\mathbf{p}_2}{2}\right) \times\exp\left[-\frac{1}{8}(\mathbf{p}_1-\mathbf{p}_2)^2R^2\right]. \quad (8)$$

For the K and η mesons, we use the S-wave harmonic oscillator wavefunction

$$\psi^{n=1;L=0}(\mathbf{p}_1,\mathbf{p}_2)=\left(\frac{R^2}{\pi}\right)^{3/4}\exp\left[-\frac{1}{8}(\mathbf{p}_1-\mathbf{p}_2)^2R^2\right], \quad (9)$$

where the parameter R denotes the meson's radius.

With the $^3P_0$ model, the general expression of the amplitudes for the decay of $0^+(2\,^3P_0)$ $c\bar{s}$ into two pseudoscalar mesons reads

$$\mathcal{M}\left(c\bar{s}(2\,^3P_0)\to 0^-+0^-\right)=\alpha\frac{\gamma\sqrt{2E_AE_BE_C}}{6\sqrt{3}}\left[I_0-2I_\pm\right], \quad (10)$$

with the spatial integral $I_\pm$ and I_0

$$I_\pm=\frac{-2\sqrt{3}}{\sqrt{5}}\frac{\pi^{1/4}\left(R_A^2\right)^{5/4}\left(R_B^2\right)^{3/4}\left(R_C^2\right)^{3/4}}{\left(R_A^2+R_B^2+R_C^2\right)^{9/2}} \times\left[10R_A^4+\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)^2-10\left(R_B^2+R_C^2\right)^2\right] \times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad (11)$$

$$\begin{aligned}I_0=&-2\sqrt{\frac{3}{5}}\frac{\pi^{1/4}\left(R_A^2\right)^{5/4}\left(R_B^2\right)^{3/4}\left(R_C^2\right)^{3/4}}{\left(R_A^2+R_B^2+R_C^2\right)^{11/2}}\\ &\times\left\{20R_A^6\left[\left(R_B^2+R_C^2\right)\mathbf{k}_B^2-2\right]\right.\\ &+2\left(R_B^2+R_C^2\right)R_A^4\left[\mathbf{k}_B^4\left(R_B^2+R_C^2\right)^2-\left(R_B^2+R_C^2\right)\mathbf{k}_B^2-20\right]\\ &+\left(R_B^2+R_C^2\right)^2R_A^2\left[\left(R_B^2+R_C^2\right)^2\mathbf{k}_B^4-32\left(R_B^2+R_C^2\right)\mathbf{k}_B^2+40\right]\\ &\left.-10\left(R_B^2+R_C^2\right)^3\left[\mathbf{k}_B^2\left(R_B^2+R_C^2\right)-4\right]\right\}\\ &\times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad (12)\end{aligned}$$

where the indices A, B and C correspond to the $0^+(2\,^3P_0)$ $c\bar{s}$ state, $D_{(s)}$ and $K(\eta)$, respectively. The factor α in (10) is 1 for the $D^0K^+(D^+K^0)$ modes and $2/\sqrt{6}$ for the $D_s^+\eta$ mode.

3.2 Double pion decays

The $0^+(2\,^3P_0)$ $c\bar{s}$ state can also decay into $D_s^*\pi\pi$ and $D_{sJ}^*(2317)\pi\pi$. Such a decay could occur via the virtual intermediate $f_0(980)$ or σ meson, which is shown in Fig. 2. So we only consider the contribution from $f_0(980)$ to make an estimate of the double pion decay. The dominant decay mode of $f_0(980)$ is $\pi\pi$. In this work, we assume that $f_0(980)$ is composed of $s\bar{s}$.

The effective Lagrangian describing $f_0(980)\to\pi\pi$ reads

$$\mathcal{L}_{f_0(980)\pi\pi}=g_{f_0\pi\pi}\left[2\pi^+\pi^-+\pi^0\pi^0\right]f_0, \quad (13)$$

where the coupling constant $g_{f_0\pi\pi}$ is taken in 0.83–1.3 GeV.

3.2.1 $c\bar{s}(2\,^3P_0)\to D_s^*f_0(980)\to D_s^*\pi\pi$

The amplitude of the $c\bar{s}(2\,^3P_0)\to D_s^*f_0(980)\to D_s^*\pi\pi$ decay chain can be expressed as

$$\mathcal{M}\left(c\bar{s}(2\,^3P_0)\to D_s^*f_0(980)\to D_s^*\pi\pi\right) =\mathcal{M}\left(c\bar{s}(2\,^3P_0)\to D_s^*f_0(980)\right)\frac{\mathrm{i}}{k^2-m_{f_0}^2}\sqrt{\lambda_{\pi\pi}}g_{f_0\pi\pi}, \quad (14)$$

with $\lambda_{\pi^+\pi^-}=2$ and $\lambda_{\pi^0\pi^0}=1$. $\mathcal{M}(c\bar{s}(2\,^3P_0)\to D_s^*f_0(980))$ can be calculated in the $^3P_0$ model using the wavefunction of $f_0(980)$:

$$\psi^{n=1;L=1}(\mathbf{p}_1,\mathbf{p}_2)=\mathrm{i}\sqrt{\frac{2}{3}}\frac{R^{5/2}}{\pi^{1/4}}\mathcal{Y}_1^m(\mathbf{p}_1-\mathbf{p}_2) \times\exp\left[-\frac{1}{8}(\mathbf{p}_1-\mathbf{p}_2)^2R^2\right]. \quad (15)$$

Its expression reads

$$\mathcal{M}\left(c\bar{s}(2\,^3P_0)\to D_s^*f_0(980)\right) =\frac{\gamma\sqrt{6E_AE_BE_C}}{9}\left[I_{0,0}^{0,0}+I_{0,0}^{1,-1}+I_{0,0}^{-1,1}+I_{0,1}^{1,0} +I_{0,1}^{0,1}+I_{0,-1}^{-1,0}+I_{0,-1}^{0,-1}\right], \quad (16)$$

where the indices A, B and C correspond to the $0^+(2\,^3P_0)$ $c\bar{s}$ state, and to D_s^* and $f_0(980)$. The expressions of $I_{M_{LB},M_{LC}}^{M_{LA},m}$ are collected in Appendix A.

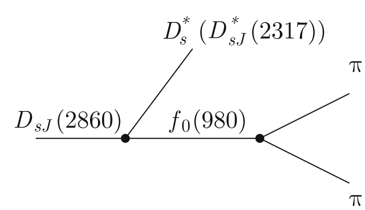


Fig. 2. The Feynman diagram for the double pion decays of the $0^+(2\,^3P_0)$ $c\bar{s}$ state

3.2.2 $c\bar{s}(2\,^3P_0) \to D^*_{sJ}(2317)f_0(980) \to D^*_{sJ}(2317)\pi\pi$

We treat $D^*_{sJ}(2317)$ as a pure $c\bar{s}(0^+)$ state. Then we can use the $^3P_0$ model to make a very rough estimate of $c\bar{s}(2\,^3P_0) \to D^*_{sJ}(2317)f_0(980) \to D^*_{sJ}(2317)\pi\pi$. Similarly, the amplitude of the $c\bar{s}(2\,^3P_0) \to D^*_{sJ}(2317)f_0(980) \to D^*_{sJ}(2317)\pi\pi$ decay chain reads

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}\big(2\,^3P_0\big) \to D^*_{sJ}(2317)f_0(980) \to D^*_{sJ}(2317)\pi\pi\big)\\&\quad= \mathcal{M}\big(c\bar{s}\big(2\,^3P_0\big) \to D^*_{sJ}(2317)f_0(980)\big)\\&\quad\times \frac{\mathrm{i}}{k^2 - m^2_{f_0}}\sqrt{\lambda_{\pi\pi}}g_{f_0\pi\pi}\,, \qquad (17)\end{aligned}$$

where

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}\big(2\,^3P_0\big) \to D^*_{sJ}(2317)f_0(980)\big)\\&= \frac{\gamma\sqrt{2E_AE_BE_C}}{9\sqrt{3}}\big[I^{0,0}_{0,0} + I^{0,1}_{0,1} + I^{0,1}_{1,0} + I^{1,0}_{0,1} + I^{1,0}_{1,0} + I^{0,-1}_{0,-1}\\&\quad + I^{0,-1}_{-1,0} + I^{-1,0}_{0,-1} + I^{-1,0}_{-1,0} + I^{1,-1}_{0,0} + I^{-1,1}_{0,0}\\&\quad + I^{0,0}_{1,-1} + I^{0,0}_{-1,1} + 2I^{1,1}_{1,1} + 2I^{-1,-1}_{-1,-1}\big]\,. \qquad (18)\end{aligned}$$

The indices A, B and C correspond to the $0^+(2\,^3P_0)$ $c\bar{s}$ state, and to $D^*_{sJ}(2317)$ and $f_0(980)$, respectively. The lengthy expressions of $I^{M_{L_A},m}_{M_{L_B},M_{L_C}}$ are collected in Appendix A.

The three-body decay width of $A \to BCD$ reads [27]

$$\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M_A^2}\int |\mathcal{M}|^2\,\mathrm{d}m^2_{12}\,\mathrm{d}m^2_{23}\,. \qquad (19)$$

4 Strong decays of the $1^-(1\,^3D_1)$ $c\bar{s}$ state

4.1 D^0K^+, D^+K^0, $D^+_s\eta$ modes

Using the harmonic oscillator wavefunction of the $1^-(1\,^3D_1)$ $c\bar{s}$ state,

$$\begin{aligned}\psi^{n=1;L=2}(\mathbf{p}_1,\mathbf{p}_2) &= \frac{R^{7/2}}{\sqrt{15}\pi^{1/4}}\mathcal{Y}^m_2\left(\frac{\mathbf{p}_1 - \mathbf{p}_2}{2}\right)\\&\quad\times \exp\left[-\frac{1}{8}(\mathbf{p}_1 - \mathbf{p}_2)^2R^2\right], \qquad (20)\end{aligned}$$

we have the decay amplitude

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}(1\,^3D_1) \to 0^- + 0^-\big)\\&\quad= \alpha\frac{\gamma\sqrt{8E_AE_BE_C}}{3}\\&\quad\times\left[-\frac{1}{\sqrt{30}}I^{0,0} + \frac{1}{2\sqrt{10}}I^{1,-1} + \frac{1}{2\sqrt{10}}I^{-1,1}\right], \qquad (21)\end{aligned}$$

where

$$\begin{aligned}I^{0,0} &= \frac{|\mathbf{k}_B|\pi^{1/4}R_A^{7/2}R_B^{3/2}R_C^{3/2}(\xi - 1)}{\big(R_A^2 + R_B^2 + R_C^2\big)^{5/2}}\\&\quad\times\big[\big(R_A^2 + R_B^2 + R_C^2\big)(\xi^2 - 1)\mathbf{k}_B^2 + 8\big]\\&\quad\times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\big(R_B^2 + R_C^2\big)}{8\big(R_A^2 + R_B^2 + R_C^2\big)}\right], \qquad (22)\\I^{1,-1} = I^{-1,1} &= -\frac{4\sqrt{3}|\mathbf{k}_B|\pi^{1/4}R_A^{7/2}R_B^{3/2}R_C^{3/2}(\xi - 1)}{\big(R_A^2 + R_B^2 + R_C^2\big)^{5/2}}\\&\quad\times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\big(R_B^2 + R_C^2\big)}{8\big(R_A^2 + R_B^2 + R_C^2\big)}\right]. \qquad (23)\end{aligned}$$

The indices A, B and C correspond to the $1^-(1\,^3D_1)$ $c\bar{s}$ state, and to $D_{(s)}$ and $K(\eta)$, respectively.

4.2 D^*K, DK^*, $D^{*+}_s\eta$ modes

The $1^-(1\,^3D_1)$ $c\bar{s}$ state can also decay into the D^*K, DK^*, $D^{*+}_s\eta$ modes. The general decay amplitude can be expressed as

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}\big(1\,^3D_1\big) \to 1^- + 0^-\big)\\&\quad= \alpha\frac{\gamma\sqrt{8E_AE_BE_C}}{3\sqrt{2}}\left[\frac{1}{\sqrt{30}}I^{0,0} + \frac{1}{2\sqrt{10}}I^{1,1} + \frac{1}{2\sqrt{10}}I^{-1,-1}\right], \qquad (24)\end{aligned}$$

where $I^{0,0}$, $I^{1,-1}$ and $I^{-1,1}$ are the same as those in (22) and (23).

4.3 Double pion decays

4.3.1 $D_{sJ}(2860) \to D^*_{sJ}(2317)f_0(980) \to D^*_{sJ}(2317)\pi\pi$

The amplitude of the $c\bar{s}(1\,^3D_1) \to D^*_{sJ}(2317)f_0(980) \to D^*_{sJ}(2317)\pi\pi$ decay chain is written as

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}\big(1\,^3D_1\big) \to D^*_{sJ}(2317)f_0(980) \to D^*_{sJ}(2317)\pi\pi\big)\\&\quad= \mathcal{M}\big(c\bar{s}(1\,^3D_1) \to D^*_{sJ}(2317)f_0(980)\big)\\&\quad\times\frac{\mathrm{i}}{k^2 - m^2_{f_0}}\sqrt{\lambda_{\pi\pi}}g_{f_0\pi\pi}\,, \qquad (25)\end{aligned}$$

where

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}(1\,^3D_1) \to D^*_{sJ}(2317)f_0(980)\big)\\&= \frac{\gamma\sqrt{8E_AE_BE_C}}{3}\\&\quad\times\left[\frac{1}{6\sqrt{10}}\big(I^{-1,0}_{-1,0} + I^{-1,0}_{0,-1} + I^{-1,1}_{0,0} + I^{1,-1}_{0,0} + I^{1,0}_{0,1} + I^{1,0}_{1,0}\big)\right.\\&\qquad + \frac{1}{3\sqrt{30}}\big(I^{0,-1}_{-1,0} + I^{0,-1}_{0,-1} + I^{0,0}_{-1,1} + I^{0,0}_{0,0}\\&\qquad\quad + I^{0,0}_{1,-1} + I^{0,1}_{0,1} + I^{0,1}_{1,0}\big)\\&\qquad\left. + \frac{1}{3\sqrt{10}}\big(I^{-1,-1}_{-1,-1} + I^{1,1}_{1,1}\big)\right]. \qquad (26)\end{aligned}$$

The indices A, B and C correspond to the $c\bar{s}(1\,^3D_1)$ state, and to $D^*_{sJ}(2317)$ and $f_0(980)$.

4.3.2 $c\bar{s}(1\,^3D_1) \to D^*_s f_0(980) \to D^*_s\pi\pi$

The amplitude of the $c\bar{s}(1\,^3D_1) \to D^*_s f_0(980) \to D^*_s\pi\pi$ decay chain is

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}\big(1\,^3D_1\big) \to D^*_s f_0(980) \to D^*_s\pi\pi\big)\\ &\quad= \mathcal{M}\big(c\bar{s}\big(1\,^3D_1\big) \to D^*_s f_0(980)\big)\frac{\mathrm{i}}{k^2-m^2_{f_0}}\sqrt{\lambda_{\pi\pi}}g_{f_0\pi\pi}\,,\end{aligned} \tag{27}$$

with

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}\big(1\,^3D_1\big) \to D^*_s f_0(980)\big)\\ &\quad= \frac{\sqrt{5}\gamma\sqrt{8E_AE_BE_C}}{3\sqrt{3}}\Bigg[\frac{\sqrt{3}}{20}\big(I^{-1,0}_{0,-1}+I^{-1,1}_{0,0}+I^{1,-1}_{0,0}+I^{1,0}_{0,1}\big)\\ &\qquad+\frac{1}{10}\big(I^{0,-1}_{0,-1}+I^{0,0}_{0,0}+I^{0,1}_{0,1}\big)\Bigg]\,,\end{aligned} \tag{28}$$

where the indices A, B and C correspond to the $c\bar{s}(1\,^3D_1)$ state, and to D^*_s and $f_0(980)$, respectively. We collect the detailed expressions of $I^{M_{L_A},m}_{M_{L_B},M_{L_C}}$ in (26) and (28) in Appendix B.

5 Strong decays of the $1^-(2\,^3S_1)$ $c\bar{s}$ state

5.1 D^0K^+, D^+K^0, $D^+_s\eta$ modes

Using the harmonic oscillator wavefunction of the $1^-(2\,^3S_1)$ $c\bar{s}$ state,

$$\begin{aligned}\psi^{n=2;L=0}(\mathbf{p}_1,\mathbf{p}_2) &= \frac{1}{\sqrt{4\pi}}\left(\frac{4R^3}{\sqrt{\pi}}\right)^{1/2}\sqrt{\frac{2}{3}}\\ &\quad\times\left[\frac{3}{2}-\frac{R^2}{4}(\mathbf{p}_1-\mathbf{p}_2)^2\right]\\ &\quad\times\exp\left[-\frac{1}{8}(\mathbf{p}_1-\mathbf{p}_2)^2R^2\right],\end{aligned}$$

the general decay amplitude of $c\bar{s}(2\,^3S_1) \to 0^- + 0^-$ can be written as

$$\mathcal{M}\big(c\bar{s}\big(2\,^3S_1\big) \to 0^-+0^-\big) = \frac{\gamma\sqrt{8E_AE_BE_C}}{\sqrt{3}}\left(\frac{1}{6}I^{0,0}_{0,0}\right),$$

where

$$\begin{aligned}I^{0,0}_{0,0} &= -\sqrt{\frac{1}{2}}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{3/2}R_B^{3/2}R_C^{3/2}}{\big(R_A^2+R_B^2+R_C^2\big)^{5/2}}\\ &\quad\times\big\{-6\big(R_A^2+R_B^2+R_C^2\big)(1+\xi)\\ &\qquad+R_A^2\big[4+20\xi+\mathbf{k}_B^2\big(R_A^2+R_B^2+R_C^2\big)\\ &\qquad\qquad\times(-1+\xi)^2(1+\xi)\big]\big\}\\ &\quad\times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\big(R_B^2+R_C^2\big)}{8\big(R_A^2+R_B^2+R_C^2\big)}\right].\end{aligned} \tag{29}$$

The indices A, B and C are for the $c\bar{s}(2\,^3S_1)$ state, and $D_{(s)}$ and $K(\eta)$, respectively.

5.2 D^*K, DK^*, $D^{*+}_s\eta$ modes

Similarly, one can get

$$\mathcal{M}\big(c\bar{s}\big(2\,^3S_1\big) \to 1^-+0^-\big) = \frac{\gamma\sqrt{8E_AE_BE_C}}{\sqrt{3}}\left(\frac{1}{3\sqrt{2}}I^{0,0}_{0,0}\right), \tag{30}$$

where the expression of $I^{0,0}_{0,0}$ is the same as that in (29).

5.3 Double pion decays

5.3.1 $c\bar{s}(2\,^3S_1) \to D^*_{sJ}(2317)f_0(980) \to D^*_{sJ}(2317)\pi\pi$

The amplitude of the $c\bar{s}(2\,^3S_1) \to D^*_{sJ}(2317)f_0(980) \to D^*_{sJ}(2317)\pi\pi$ decay chain is

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}\big(2\,^3S_1\big) \to D^*_{sJ}(2317)f_0(980) \to D^*_{sJ}(2317)\pi\pi\big)\\ &\quad= \mathcal{M}\big(c\bar{s}\big(2\,^3S_1\big) \to D^*_{sJ}(2317)f_0(980)\big)\\ &\quad\times\frac{\mathrm{i}}{k^2-m^2_{f_0}}\sqrt{\lambda_{\pi\pi}}g_{f_0\pi\pi}\,,\end{aligned} \tag{31}$$

with

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}\big(2\,^3S_1\big) \to D^*_{sJ}(2317)f_0(980)\big)\\ &\quad= \frac{\gamma\sqrt{8E_AE_BE_C}}{\sqrt{3}}\Bigg[-\frac{1}{18}\big(I^{0,-1}_{-1,0}+I^{0,-1}_{0,-1}+I^{0,0}_{-1,1}+I^{0,0}_{0,0}\\ &\qquad+I^{0,0}_{1,-1}+I^{0,1}_{0,1}+I^{0,1}_{1,0}\big)\Bigg]\,,\end{aligned} \tag{32}$$

where the indices A, B and C correspond to the $c\bar{s}(2\,^3S_1)$ state, and to $D^*_{sJ}(2317)$ and $f_0(980)$.

5.3.2 $c\bar{s}(2\,^3S_1) \to D^*_s f_0(980)) \to D^*_s\pi\pi$

The amplitude of the $c\bar{s}(2\,^3S_1) \to D^*_s f_0(980) \to D^*_s\pi\pi$ decay chain is

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}\big(2\,^3S_1\big) \to D^*_s f_0(980) \to D^*_s\pi\pi\big)\\ &\quad= \mathcal{M}\big(c\bar{s}\big(2\,^3S_1\big) \to D^*_s f_0(980)\big)\frac{\mathrm{i}}{k^2-m^2_{f_0}}\sqrt{\lambda_{\pi\pi}}g_{f_0\pi\pi}\,,\end{aligned} \tag{33}$$

with

$$\begin{aligned}&\mathcal{M}\big(c\bar{s}\big(2\,^3S_1\big) \to D^*_s f_0(980)\big)\\ &\quad= \frac{\gamma\sqrt{8E_AE_BE_C}}{3}\left[\frac{1}{2\sqrt{3}}\big(I^{0,-1}_{0,-1}+I^{0,0}_{0,0}+I^{0,1}_{0,1}\big)\right],\end{aligned} \tag{34}$$

where the indices A, B and C correspond to the $c\bar{s}(2\,^3S_1)$ state, and to D^*_s and $f_0(980)$. We collect the expressions of $I^{M_{L_A},m}_{M_{L_B},M_{L_C}}$ in (32)) and (34) in Appendix C.

6 Strong decays of the $3^-(1^3D_3)$ $c\bar{s}$ state

6.1 D^0K^+, D^+K^0, $D_s\eta$ modes

The general expression of the decay amplitude for $c\bar{s}(1\,^3D_3) \to 0^- + 0^-$ reads

$$\mathcal{M}\big(c\bar{s}(1\,^3D_3) \to 0^- + 0^-\big) = \alpha\frac{\gamma\sqrt{8E_AE_BE_C}}{\sqrt{21}}\left[\frac{1}{2\sqrt{5}}I^{0,0} + \frac{1}{2\sqrt{15}}I^{1,-1} + \frac{1}{2\sqrt{15}}I^{-1,1}\right], \quad (35)$$

where the expressions of $I^{0,0}$, $I^{1,-1}$ and $I^{-1,1}$ are also the same as those in (22) and (23). The indices of A, B and C correspond to the $3^-(1\,^3D_3)$ $c\bar{s}$ state, and to $D_{(s)}$ and $K(\eta)$, respectively.

6.2 D^*K, DK^*, $D_s^{*+}\eta$ modes

The general decay amplitude for $c\bar{s}(1\,^3D_3) \to 1^- + 0^-$ can be expressed as

$$\mathcal{M}\big(c\bar{s}(1\,^3D_3) \to 1^- + 0^-\big) = \alpha\frac{\gamma\sqrt{8E_AE_BE_C}}{\sqrt{42}}\left[\frac{2}{\sqrt{30}}I^{0,0} + \frac{1}{3}\sqrt{\frac{2}{5}}I^{1,-1} + \frac{1}{3}\sqrt{\frac{2}{5}}I^{-1,1}\right], \quad (36)$$

where the expressions of $I^{0,0}$, $I^{1,-1}$ and $I^{-1,1}$ are the same as those in (22) and (23).

7 Numerical results

The calculation of the transition amplitude using the $^3P_0$ model involves two parameters: the strength of quark pair creation from the vacuum γ and the R value in the harmonic oscillator wavefunction. We follow the convention of [28] and take $\gamma = 6.9$, which is $\sqrt{96\pi}$ times larger than that used by other groups [29, 30]. γ is a universal parameter in the $^3P_0$ model.

Throughout our calculation of the decay amplitudes, we use the harmonic oscillator wavefunctions as commonly done in the framework of the $^3P_0$ model in the literature, which indeed brings about some uncertainties. A more realistic wavefunction like that from the linear potential model seems more reliable in describing the properties of the meson. But the decay amplitude with more realistic wavefunctions does not have a simple analytical expression. Moreover, such a more complicated calculation with more realistic wavefunctions does not always lead to systematic improvements because of the inherent uncertainties of the $^3P_0$ strong decay model itself, as discussed in [17]. The value of the parameter R in the harmonic oscillator wavefunction can be fixed to reproduce the realistic root mean square (RMS) radius, which is obtained by solving the Schrödinger equation with the linear potential.

The R values in the harmonic oscillator wavefunctions are taken from [29]. We collect all these values in Table 1. The masses of $K^{(*)}$, η, $f_0(980)$ and of the charm–strange mesons are taken from PDG [27].

The dependence of $D_{sJ}(2860)$'s total two-body decay widths on its R_A and its possible quantum numbers is presented in Fig. 3. As expected, its width vanishes around $R_A = 2.4\,\text{GeV}^{-1}$ for the $0^+(2\,^3P_0)$ case because of the node in the radial wavefunction.

The variation of the decay widths of $D_{sJ}(2860)$'s and $D_{sJ}(2715)$'s different modes with R_A and their quantum numbers is shown in Fig. 4.

The variation of $D_{sJ}(2860)$'s three-body decay widths with R_A is presented in Fig. 5.

The variation of $D_{sJ}(2715)$'s different three-body decay widths with R_A is presented in Fig. 6.

We present the variation of both $D_{sJ}(2860)$'s and $D_{sJ}(2715)$'s two-body decay widths with R_A as either a $1\,^3D_1$ state or a $2\,^3S_1$ state in Fig. 7.

We take $R_A = 3.1\,\text{GeV}^{-1}$ for $0^+(2\,^3P_1)$, $R_A = 3.2\,\text{GeV}^{-1}$ for the $1^-(2\,^3S_1)$ $c\bar{s}$ states, and $R_A = 2.94\,\text{GeV}^{-1}$ for the $1^-(1\,^3D_1)$ and the $3^-(1\,^3D_3)$ $c\bar{s}$ states to estimate the decay widths, and we collect them in Table 2, which is the main result of this work.

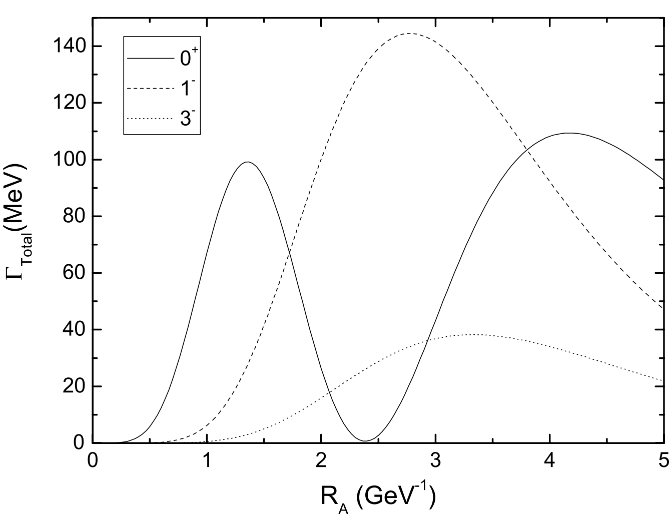


Fig. 3. The dependence of $D_{sJ}(2860)$'s total two-body decay widths on R_A and on possible quantum numbers

Table 1. The meson masses and R values used in our calculation

	K	K^*	η	$f_0(980)$	$D(0^-)$	$D_s(0^-)$	$D^*(1^-)$	$D_s^*(1^-)$	$D_{sJ}^*(2317)$
Mass (GeV)	0.494	0.892	0.548	0.980	1.869	1.968	2.007	2.112	2.317
R (GeV^{-1})	2.17	3.13	2.08	2.78	2.33	1.92	2.70	2.22	2.70

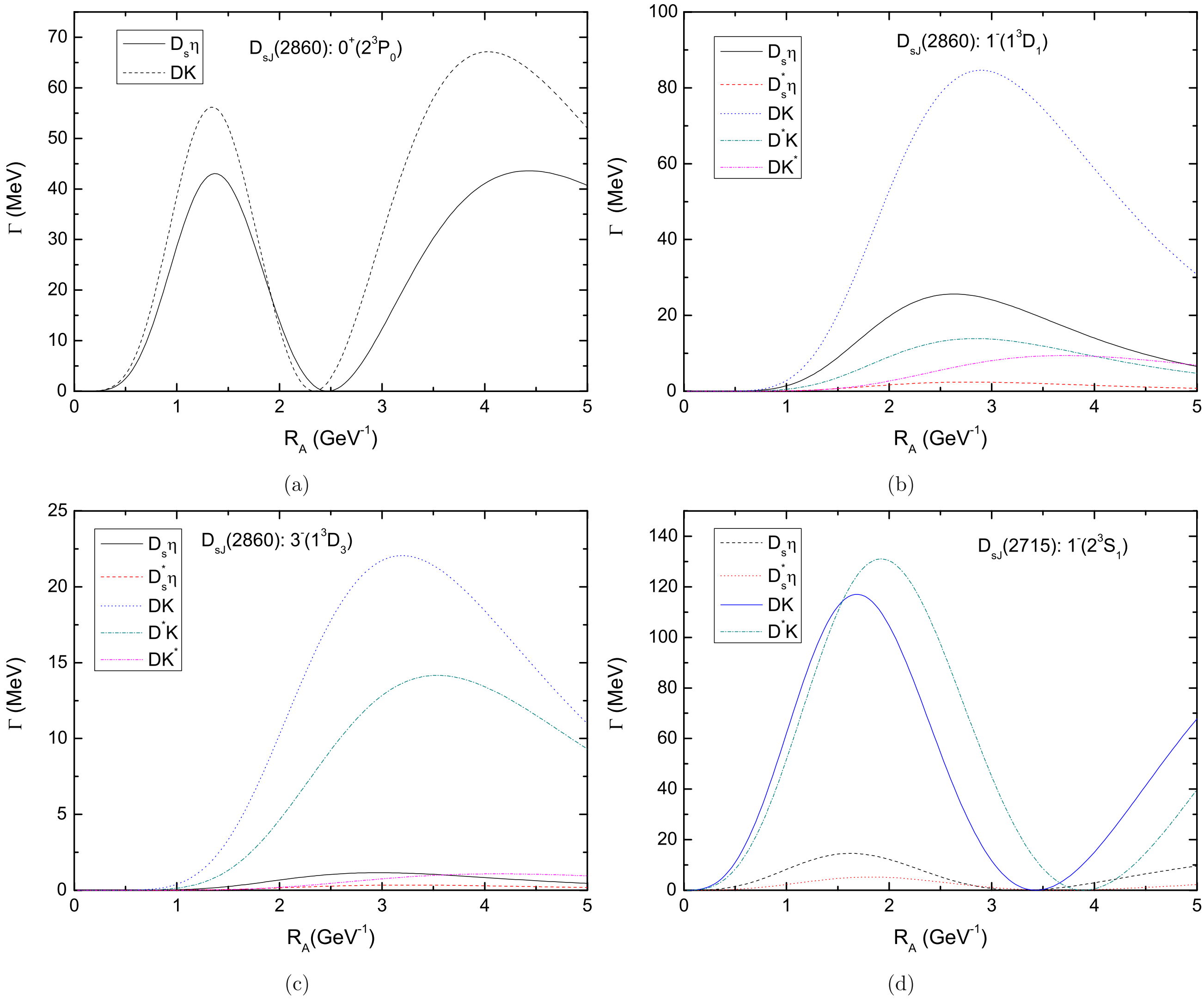


Fig. 4. a The dependence of $D_{sJ}(2860)$'s two-body decay widths on R_A as a $0^+(2\,^3P_0)$ candidate; **b** for the case of $1^-(1\,^3D_1)$; **c** for the case of $3^-(1\,^3D_3)$; **d** for the case of $D_{sJ}(2715)$ as a $1^-(2\,^3S_1)$ candidate

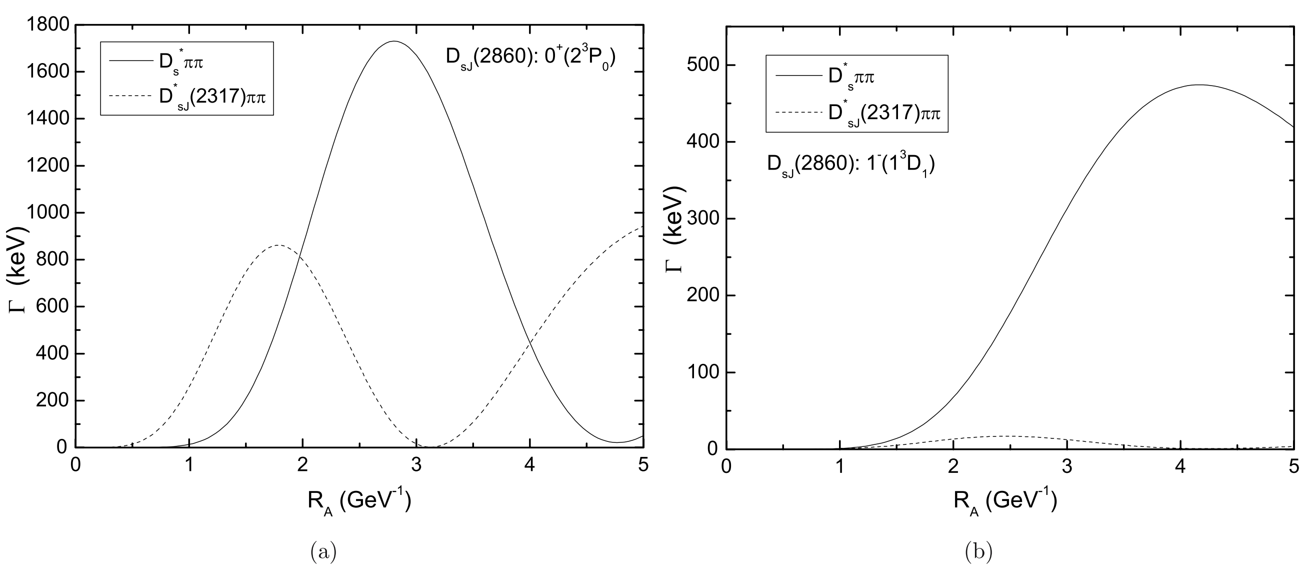
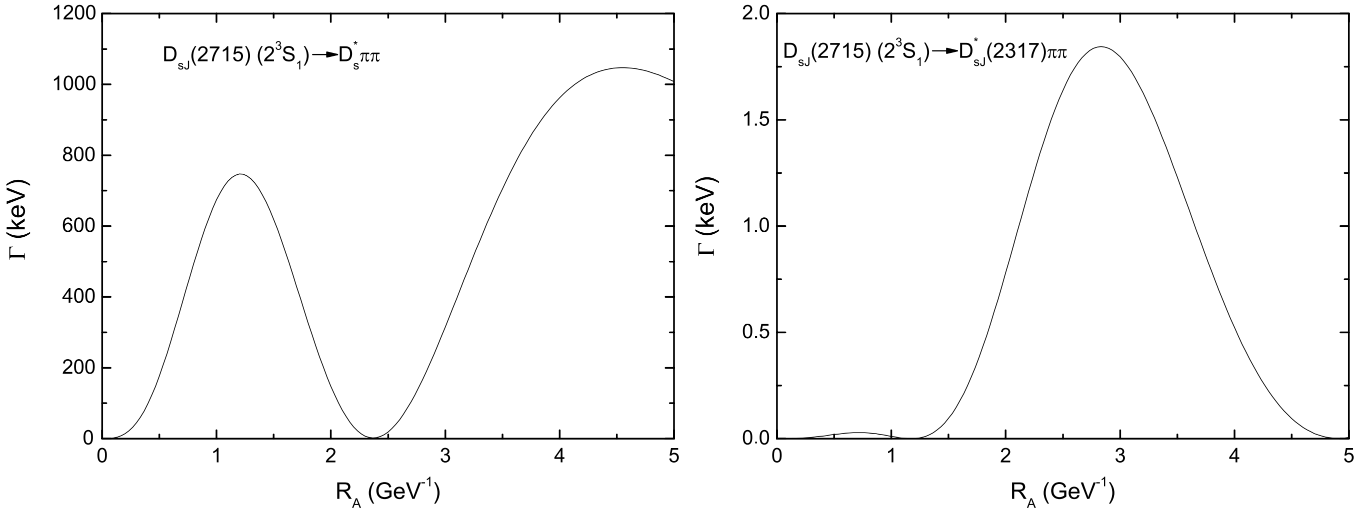
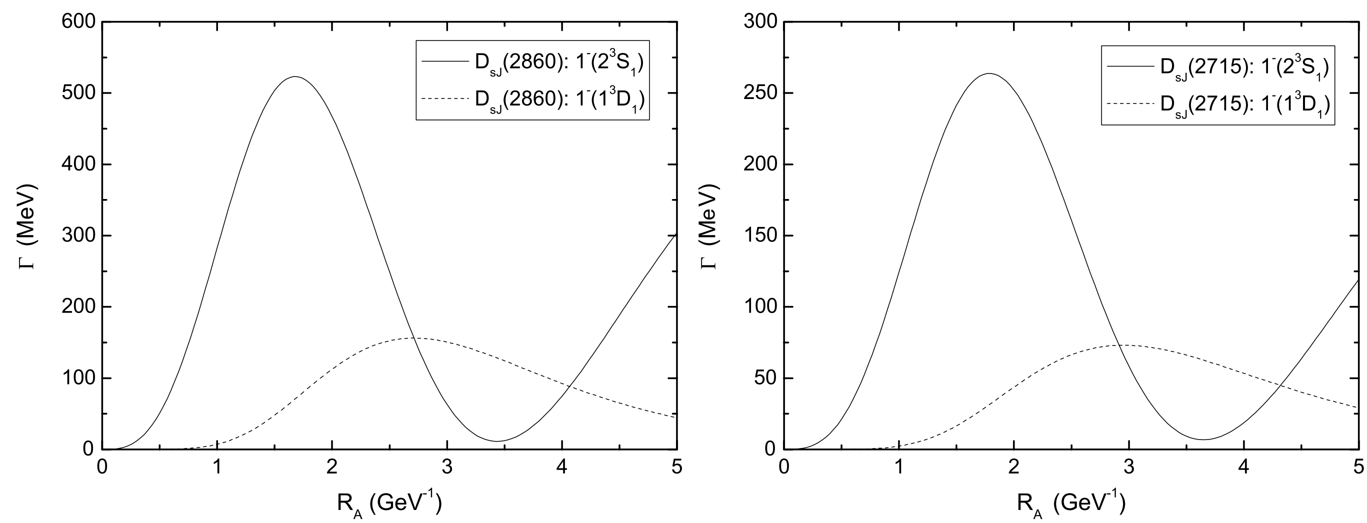


Fig. 5. a The dependence of $D_{sJ}(2860)$'s three-body decay widths on R_A as a 0^+ $c\bar{s}$ state; **b** as a $1^-(\,^3D_1)$ $c\bar{s}$ state



(a) (b)

Fig. 6. $D_{sJ}(2715)$'s three-body decay widths as $2\,^3S_1$ $c\bar{s}$



(a) (b)

Fig. 7. The variation of both $D_{sJ}(2860)$'s and $D_{sJ}(2715)$'s two-body decay widths with R_A as either a $1\,^3D_1$ state or a $2\,^3S_1$ state

Table 2. The decay widths of $D_{sJ}(2860)$ and $D_{sJ}(2715)$ as different $c\bar{s}$ candidates. The two-body decay width and the total width are in units of MeV, while the three-body decay width is in units of keV

	$D_{sJ}(2860)$							
	DK	$D_s\eta$	D^*K	$D_s^*\eta$	DK^*	$D_s^*\pi\pi$	$D_{sJ}^*(2317)\pi\pi$	Total
$0^+(2\,^3P_0)$	37	16	–	–	–	1600	2	54
$1^-(1\,^3D_1)$	84	24	14	2	7.8	313	13	132
$1^-(2\,^3S_1)$	0.012	0.104	24	1.6	64	923	30	90
$3^-(1\,^3D_3)$	22	1.2	13	0.3	0.71	–	–	37
	$D_{sJ}(2715)$							
	DK	$D_s\eta$	D^*K	$D_s^*\eta$		$D_s^*\pi\pi$	$D_{sJ}^*(2317)\pi\pi$	Total
$1^-(2^3S_1)$	3.2	0.05	27.2	0.54		477	1.63	32
$1^-(1^3D_1)$	49.4	13.2	8	2.4		49	0.5	73

8 Discussion

We have studied the strong decay patterns of $D_{sJ}(2860)$ and $D_{sJ}(2715)$ using the $^3P_0$ model, assuming that they are excited $c\bar{s}$ candidates. If $D_{sJ}(2715)$ is the $1^-(2\,^3S_1)$ candidate, D^*K should be the dominant decay mode with a width around 27 MeV. In contrast, the decay width of the DK mode is only 3 MeV. The $D_s^*\pi\pi$ mode has a rather large width around 0.48 GeV. Its total width is only 32 MeV, much smaller than the experimental value $\Gamma = (115\pm 20^{+36}_{-32})$ MeV. However, one should be cautious that the total width is sensitive to R_A and the node position. On the other hand, DK becomes the dominant decay mode, and its total width is around 73 MeV, roughly consistent with the experimental data if one identifies $D_{sJ}(2715)$ as the $1^-(1\,^3D_1)$ $c\bar{s}$ state. With this assignment, the $D_s\eta$ mode has a width of 13 MeV and becomes significant. It will be helpful to search for this mode experimentally.

If $D_{sJ}(2860)$ is the $1^-(2\,^3S_1)$ candidate, the dominant decay modes are DK^* and D^*K. The decay width of the DK mode is only 12 keV, which is tiny. Moreover, its mass is much higher than the quark model prediction (2658 ± 15) MeV [31]. Its total width is around 90 MeV, significantly larger than the experimental value $\Gamma = (48\pm 7\pm 10)$ MeV. Similarly, the total width becomes 132 MeV if $D_{sJ}(2860)$ is the $1^-(1\,^3D_1)$ candidate. The ratio of its two-body decay modes is $\Gamma(DK):\Gamma(D_s\eta):\Gamma(D^*K):\Gamma(D_s^*\eta):\Gamma(DK^*)\approx 22:1:13:0.3:0.7$. We are inclined to conclude that these two assignments are not favorable for $D_{sJ}(2860)$, based on the available experimental information.

f $D_{sJ}(2860)$ is the $0^+(2\,^3P_0)$ $c\bar{s}$ state, its total width is around 54 MeV, roughly consistent with the experimental value considering the theoretical uncertainty. The dominant mode is DK with a width of 37 MeV. Also, the $D_s\eta$ mode has the very large width of 16 MeV. The $D_s^*\pi\pi$ three-body mode has the quite large width of 1.6 MeV. At present only the DK decay mode is observed experimentally.

If $D_{sJ}(2860)$ is the $3^-(1\,^3D_3)$ $c\bar{s}$ state, its total width is 37 MeV, also compatible with the experimental data. Its dominant decay mode is DK. But now the D^*K mode has the large width of 13 MeV. The ratio of its two-body decay modes is $\Gamma(DK):\Gamma(D_s\eta):\Gamma(D^*K):\Gamma(D_s^*\eta):\Gamma(DK^*)\approx 22:1:13:0.3:0.7$.

Naively one would expect both the $0^+(2\,^3P_0)$ and $3^-(1\,^3D_3)$ $c\bar{s}$ states to lie around 2860 MeV. The available experimental information is not enough to distinguish these possibilities. However, the $0^+(2\,^3P_0)$ $c\bar{s}$ state does not decay into DK^*, D^*K and $D_s^*\eta$. The $3^-(1\,^3D_3)$ $c\bar{s}$ state does not have a large $D_s\eta$ decay width. Therefore, the experimental search of $D_{sJ}(2860)$ in the channels $D_s\eta$, DK^*, D^*K and $D_s^*\eta$ is strongly called for.

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Appendix A: The $0^+(2\,^3P_0)$ case

A.1 The expressions of $I^{0,0}_{0,0}$, $I^{1,-1}_{0,0}$, $I^{-1,1}_{0,0}$, $I^{0,1}_{0,1}$, $I^{0,-1}_{0,-1}$, $I^{1,0}_{0,1}$ and $I^{-1,0}_{0,-1}$ for $c\bar{s}(2\,^3P_0)\to D_s^* f_0(980)\to D_s^*\pi\pi$

We have

$$\begin{aligned}
&I^{0,0}_{0,0} = \\
&-4\sqrt{\frac{6}{5}}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{5/2}R_B^{3/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}} \\
&\times\Bigg\{R_A^2\Bigg[\frac{1}{16}\left(R_A^2+R_B^2+R_C^2\right)^2(\xi-1)^3\xi(\xi+1)\mathbf{k}_B^4 \\
&+\frac{1}{2}\left(R_A^2+R_B^2+R_C^2\right)(6\xi^3-6\xi^2-\xi+1)\mathbf{k}_B^2+21\xi-6\Bigg] \\
&-\frac{5}{8}\left(R_A^2+R_B^2+R_C^2\right)\xi\left[\left(R_A^2+R_B^2+R_C^2\right)(\xi^2-1)\mathbf{k}_B^2+12\right]\Bigg\} \\
&\times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \qquad (A.1)
\end{aligned}$$

$$\begin{aligned}
&I^{1,-1}_{0,0} = I^{-1,1}_{0,0} = \\
&\sqrt{\frac{6}{5}}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{5/2}R_B^{3/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}} \\
&\times\Big\{\mathbf{k}_B^2(\xi-1)^2\xi R_A^4 \\
&\quad+R_A^2\left[\left(\mathbf{k}_B^2\left(R_B^2+R_C^2\right)(\xi-1)^2+18\right)\xi-8\right] \\
&\quad-10\left(R_B^2+R_C^2\right)\xi\Big\} \\
&\times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \qquad (A.2)
\end{aligned}$$

$$\begin{aligned}
&I^{0,1}_{0,1} = I^{0,-1}_{0,-1} = \\
&-\sqrt{\frac{6}{5}}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{5/2}R_B^{3/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}} \\
&\times\Big\{\left[R_A^2\left(\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(\xi-1)^2+18\right)\right. \\
&\quad\left.-10\left(R_B^2+R_C^2\right)\right](\xi-1)\Big\} \\
&\times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \qquad (A.3)
\end{aligned}$$

$$\begin{aligned}
&I^{1,0}_{0,1} = I^{-1,0}_{0,-1} = \\
&-4\sqrt{\frac{6}{5}}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{5/2}R_B^{3/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}} \\
&\times\Bigg\{R_A^2\left[\frac{1}{4}\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(\xi+1)(\xi-1)^2+7\xi+3\right] \\
&\quad-\frac{5}{2}\left(R_A^2+R_B^2+R_C^2\right)(\xi+1)\Bigg\} \\
&\times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \qquad (A.4)
\end{aligned}$$

with $\xi=\frac{R_A^2}{R_A^2+R_B^2+R_C^2}$, where the indices A, B and C correspond to $D_{sJ}(2860)$, D_s^* and $f_0(980)$.

A.2 The expressions of $I^{M_{L_A},m}_{M_{L_B},M_{L_C}}$ appearing in the calculation of $c\bar{s}(2\,^3P_0)\to D^*_{sJ}(2317)f_0(980)\to D^*_{sJ}(2317)\pi\pi$

We have

$$I^{0,0}_{0,0} = -16\sqrt{\frac{3}{5}}\frac{\pi^{1/4}R_A^{5/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{9/2}} \times\Bigg\{\frac{1}{64}R_A^2\Big[\left(R_A^2+R_B^2+R_C^2\right)^3(\xi-1)^3\xi^2(\xi+1)\mathbf{k}_B^6 +4\left(R_A^2+R_B^2+R_C^2\right)^2(17\xi^4-20\xi^3-2\xi^2+6\xi-1)\mathbf{k}_B^4 +16\left(R_A^2+R_B^2+R_C^2\right)(57\xi^2-30\xi-2)\mathbf{k}_B^2+1344\Big] -\frac{5}{2}\left(R_A^2+R_B^2+R_C^2\right)\left[\frac{1}{16}\left(R_A^2+R_B^2+R_C^2\right)^2\xi^2(\xi^2-1)\mathbf{k}_B^4 +\frac{1}{4}\left(R_A^2+R_B^2+R_C^2\right)(6\xi^2-1)\mathbf{k}_B^2+3\right]\Bigg\} \times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \tag{A.5}$$

$$I^{0,1}_{0,1}=I^{0,1}_{1,0}=I^{0,-1}_{0,-1}=I^{0,-1}_{-1,0}= -16\sqrt{\frac{3}{5}}\frac{\pi^{1/4}R_A^{5/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{9/2}}\Bigg\{R_A^2\Bigg[\frac{1}{16}\left(R_A^2+R_B^2+R_C^2\right) \times\left(\xi\left[\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(\xi-1)^3+40\xi-52\right]+12\right)\mathbf{k}_B^2+7\Bigg] -\frac{5}{8}\left(R_A^2+R_B^2+R_C^2\right)\left[\left(R_A^2+R_B^2+R_C^2\right)(\xi-1)\xi\mathbf{k}_B^2+4\right]\Bigg\} \times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \tag{A.6}$$

$$I^{1,0}_{0,1}=I^{1,0}_{1,0}=I^{-1,0}_{0,-1}=I^{-1,0}_{-1,0}= -16\sqrt{\frac{3}{5}}\frac{\pi^{1/4}R_A^{5/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{9/2}}\Bigg\{R_A^2\Bigg[\frac{1}{16}\left(R_A^2+R_B^2+R_C^2\right) \times\left(\xi\left[\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(\xi+1)(\xi-1)^2 +40\xi+4\right]-4\right)\mathbf{k}_B^2+7\Bigg] -\frac{5}{8}\left(R_A^2+R_B^2+R_C^2\right)\left[\left(R_A^2+R_B^2+R_C^2\right)(\xi+1)\xi\mathbf{k}_B^2+4\right]\Bigg\} \times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \tag{A.7}$$

$$I^{1,-1}_{0,0}=I^{-1,1}_{0,0}= 16\sqrt{\frac{3}{5}}\frac{\pi^{1/4}R_A^{5/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{9/2}}\Bigg\{R_A^2\Bigg[\frac{1}{16}\left(R_A^2+R_B^2+R_C^2\right) \times\left(\xi\left(\left[\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(\xi-1)^2+40\right]\xi-24\right)+4\right)\mathbf{k}_B^2+7\Bigg] -\frac{5}{8}\left(R_A^2+R_B^2+R_C^2\right)\left[\left(R_A^2+R_B^2+R_C^2\right)\xi^2\mathbf{k}_B^2+4\right]\Bigg\} \times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8(R_A^2+R_B^2+R_C^2)}\right], \tag{A.8}$$

$$I^{0,0}_{1,-1}=I^{0,0}_{-1,1}= 16\sqrt{\frac{3}{5}}\frac{\pi^{1/4}R_A^{5/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{9/2}}\Bigg\{R_A^2\Bigg[\frac{1}{16}\left(R_A^2+R_B^2+R_C^2\right) \times\left(\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(\xi+1)(\xi-1)^3 +8(5\xi+2)(\xi-1)\right)\mathbf{k}_B^2+7\Bigg] -\frac{5}{8}\left(R_A^2+R_B^2+R_C^2\right)\left[\left(R_A^2+R_B^2+R_C^2\right)(\xi^2-1)\mathbf{k}_B^2+4\right]\Bigg\} \times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \tag{A.9}$$

$$I^{1,1}_{1,1}=I^{-1,-1}_{-1,-1}= -8\sqrt{\frac{3}{5}}\frac{\pi^{1/4}R_A^{5/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{9/2}} \times\left\{R_A^2\left[\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(\xi-1)^2+18\right] -10\left(R_B^2+R_C^2\right)\right\}. \tag{A.10}$$

Appendix B: The $1^-(1\,^3D_1)$ case

B.1 The relevant expressions in the calculation of $c\bar{s}(1\,^3D_1)\to D^*_{sJ}(2317)f_0(980)\to D^*_{sJ}(2317)\pi\pi$

We have

$$I^{-2,0}_{-1,-1}=I^{2,0}_{1,1}= -8\sqrt{6}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{7/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}}(1+\xi) \times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \tag{B.1}$$

$$I^{-2,1}_{-1,0}=I^{-2,1}_{0,-1}=I^{2,-1}_{0,1}=I^{2,-1}_{1,0}= 8\sqrt{6}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{7/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}}\xi\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \tag{B.2}$$

$$I^{-1,-1}_{-1,-1}=I^{-1,1}_{-1,1}=I^{-1,1}_{1,-1}=I^{1,-1}_{-1,1}=I^{1,-1}_{1,-1}=I^{1,1}_{1,1}= -16\sqrt{3}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{7/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}}(-1+\xi) \times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \tag{B.3}$$

$$I^{-1,0}_{-1,0}=I^{-1,0}_{0,-1}=I^{1,0}_{0,1}=I^{1,0}_{1,0}= -2\sqrt{3}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{7/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}}\xi \times\left\{12+\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(-1+\xi^2)\right\} \times\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \tag{B.4}$$

$$I_{0,0}^{-1,1} = I_{0,0}^{1,-1} = 2\sqrt{3}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{7/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}}\left\{-4+12\xi+\mathbf{k}_B^2R_A^2\xi(-1+\xi)\right\} \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad \text{(B.5)}$$

$$I_{-1,0}^{0,-1} = I_{0,-1}^{0,-1} = I_{0,1}^{0,1} = I_{1,0}^{0,1} = -2\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{7/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}}\left\{-8+4\xi+\mathbf{k}_B^2R_A^2(-1+\xi)^2\right\} \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad \text{(B.6)}$$

$$I_{-1,1}^{0,0} = I_{1,-1}^{0,0} = 2\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{7/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}} \times\left\{4(-3+\xi)+\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(-1+\xi)^2(1+\xi)\right\} \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad \text{(B.7)}$$

$$I_{0,0}^{0,0} = -\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{7/2}R_B^{5/2}R_C^{5/2}}{2\left(R_A^2+R_B^2+R_C^2\right)^{7/2}} \times\left\{64(-1+3\xi)+\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(-1+\xi) \times\left[-4+\mathbf{k}_B^2\frac{R_A^4(-1+\xi^2)}{R_A^2+R_B^2+R_C^2}+4\xi(2+9\xi)\right]\right\} \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right]. \quad \text{(B.8)}$$

B.2 The relevant expressions in the calculation of $c\bar{s}(1\,^3D_1) \to D_s^* f_0(980) \to D_s^*\pi\pi$

We have

$$I_{0,-1}^{-2,1} = I_{0,1}^{2,-1} = 16\sqrt{3}\frac{\pi^{1/4}R_A^{7/2}R_B^{3/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}}\exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad \text{(B.9)}$$

$$I_{0,-1}^{-1,0} = I_{0,1}^{1,0} = -2\sqrt{6}\frac{\pi^{1/4}R_A^{7/2}R_B^{3/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}} \times\left[4+\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(-1+\xi^2)\right] \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad \text{(B.10)}$$

$$I_{0,0}^{-1,1} = I_{0,0}^{1,-1} = 2\sqrt{6}\frac{\pi^{1/4}R_A^{7/2}R_B^{3/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}}\left[4+\mathbf{k}_B^2R_A^2(-1+\xi)\right] \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad \text{(B.11)}$$

$$I_{0,-1}^{0,-1} = I_{0,1}^{0,1} = -2\sqrt{2}\frac{\pi^{1/4}R_A^{7/2}R_B^{3/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}} \times\left[-4+\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(-1+\xi)^2\right] \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad \text{(B.12)}$$

$$I_{0,0}^{0,0} = -\frac{1}{\sqrt{2}}\frac{\pi^{1/4}R_A^{7/2}R_B^{3/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}} \times\left\{32+\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(-1+\xi) \times\left[4+20\xi+\mathbf{k}_B^2R_A^2(-1+\xi^2)\right]\right\} \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right]. \quad \text{(B.13)}$$

Appendix C: The $1^-(2\,^3S_1)$ case

C.1 The expressions of $I_{M_{L_B},M_{L_C}}^{M_{L_A},m}$ in (32)

We have

$$I_{-1,0}^{0,-1} = I_{0,-1}^{0,-1} = I_{0,1}^{0,1} = I_{1,0}^{0,1} = \sqrt{2}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{3/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}} \times\left\{-6R_A^2+R_A^2\left[-8+28\xi+\mathbf{k}_B^2R_A^2(-1+\xi)^2\right]\right\} \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad \text{(C.1)}$$

$$I_{-1,1}^{0,0} = I_{1,-1}^{0,0} = -\sqrt{2}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{3/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}}\left\{-6\left(R_A^2+R_B^2+R_C^2\right)(1+\xi) + R_A^2\left[12+28\xi+\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(-1+\xi)^2(1+\xi)\right]\right\} \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad \text{(C.2)}$$

$$I_{0,0}^{0,0} = \frac{1}{2\sqrt{2}}\frac{|\mathbf{k}_B|\pi^{1/4}R_A^{3/2}R_B^{5/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}} \times\left\{-6\mathbf{k}_B^2R_A^4(1+\xi)-24\left(R_A^2+R_B^2+R_C^2\right)(1+3\xi) + R_A^2\left[-16+336\xi+\mathbf{k}_B^4R_A^4(-1+\xi)^2(1+\xi) + 4\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(-1+2\xi)(1+2\xi)(-1+3\xi)\right]\right\} \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right]. \quad \text{(C.3)}$$

C.2 The expressions of $I_{M_{L_B},M_{L_C}}^{M_{L_A},m}$ in (34)

We have

$$I_{0,-1}^{0,-1} = I_{0,1}^{0,1} = 2\frac{\pi^{1/4}R_A^{3/2}R_B^{3/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}}\left\{-6\left(R_A^2+R_B^2+R_C^2\right) + R_A^2\left[20+\mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)(-1+\xi)^2\right]\right\} \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right], \quad \text{(C.4)}$$

$$I_{0,0}^{0,0} = \frac{1}{2}\frac{\pi^{1/4}R_A^{3/2}R_B^{3/2}R_C^{5/2}}{\left(R_A^2+R_B^2+R_C^2\right)^{7/2}}\left\{80R_A^2-24\left(R_A^2+R_B^2+R_C^2\right) + \mathbf{k}_B^2\left(R_A^2+R_B^2+R_C^2\right)\left[-6R_A^2(1+\xi) + R_A^2\left(-4+R_A^2\mathbf{k}_B^2(-1+\xi)^2(1+\xi)+4\xi(-1+8\xi)\right)\right]\right\} \times \exp\left[-\frac{\mathbf{k}_B^2R_A^2\left(R_B^2+R_C^2\right)}{8\left(R_A^2+R_B^2+R_C^2\right)}\right]. \quad \text{(C.5)}$$

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